UNIT 7: CONIC SECTIONS
UNIT PLAN

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How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS
Conic sections are the curves that are formed when a plane cuts a cone, as shown in the figure. For example, if a cone is cut horizontally, the cross section is a circle. So, a circle is a conic section. Other ways of cutting a cone produce ellipses, parabolas, and hyperbolas. In this unit, you will find that for each conic (except the circle), there will be a lesson on how to generate the conic using its definition. Our goal in this unit is to find equations whose graphs are conic sections. We will find such equations by analyzing the geometric properties of conic sections. These properties make conic sections useful for many real-world applications. For instance, a reflecting surface with parabolic cross sections concentrates light at a single point, a medical device called a lithotripter uses elliptical reflectors to break up kidney stones by generating sound waves, and more.

CONTENT STANDARDS

Below are the standards addressed in this unit.

Texas Essential Knowledge and Skills (TEKS)	
Knowledge and Skills	Student Expectations (SE)
(3) Relations and Geometric Reasoning The student uses the process standards in mathematics to **model** and make **connections** between **algebraic** and **geometric relations**.	**(3.F)** Determine the conic section formed when a plane intersects a double-napped cone. **(3.G)**[2] Make connections between the locus definition of conic sections and their equations in rectangular coordinates; [2]This standard will also be tagged for content on parabolas. **(3.H)** Use the characteristics of an ellipse to write and apply[3] the equation of an ellipse with center (h, k). **(3.I)** Use the characteristics of a hyperbola to write and apply[3] the equation of a hyperbola with center (h, k). [3]Included to support AP readiness goals.

Focus on Disciplinary Literacy	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓			✓			✓	✓		✓	
makes use of graphic organizers	Graphic Organizers	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives	✓		✓	✓	✓					✓	
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓		✓	✓	✓	✓	✓	✓	✓	✓	
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports		✓		✓	✓	✓	✓	✓		✓	✓
includes strategies that support language development												
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓	✓		✓		✓		✓	
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share					✓		✓		✓		✓
includes sentence stems to support students with explanations	- Sentence Stems											
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics											
includes websites or equipment that enhances the lesson	Technological Support	✓		✓	✓	✓	✓		✓	✓	✓	✓
content can be presented in different forms												
uses hands-on tools or manipulatives to represent the math	- Concrete											
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

ROADMAP

AT A GLANCE: Unit 7 – Conic Sections			
Day	Date	Lesson	Lesson Title
1		1	Introduction to Conics
2		2	Geometry of a Parabola
3		3	Writing and Graphing Equations of a Parabola
4			*Unit 7 Success Day Alpha – Review Parabolas as Conic Sections*
5		4	Reflective Property of Parabolas & Other Applications
6		5	Geometry of an Ellipse
7		6	Writing and Graphing Equations of an Ellipse
8		7	Reflective Property of Ellipses & Other Applications
9		8	Geometry of a Hyperbola
10		9	Writing and Graphing Equations of a Hyperbola
11		10	Reflective Property of Hyperbolas & Other Applications
12		11	Eccentricity and Review
13			*Unit 7 Success Day Beta – Review topics based on your data*
14			End of Unit 7 Assessment

Date: _____
Lesson 1: Introduction to Conics

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **(3.F)** Determine the conic section formed when a plane intersects a double-napped cone.

◆ **(3.G)** Make connections between the locus definition of conic sections and their equations in rectangular coordinates.

Important Vocabulary

▪ Conic Section
▪ Degenerate Conic
▪ Circle
▪ Ellipse
▪ Hyperbola
▪ Parabola
▪ General Form
▪ Axis of Symmetry

Notes for Intellectual Preparation & Lesson Planning

<u>Necessary Materials and Pre-Lesson Prep</u>
▪ Graphing calculators
▪ GeoGebra activity
▪ Document camera
▪ Desmos Activity

Lesson Structure:
- 🟥 Do Now (5 min)
- 🟦 INM (35 min)
- 🟩 Student Practice (10 min)
- 🟪 Debrief (2 min)
- 🟪 Exit Ticket (3 min)

<u>Mathematical Goal of this Lesson</u>
This lesson introduces conic sections. The Do Now will have students recall basics of geometry that will be used in this lesson. This lesson contains two links to activities that allows students to see and conceptualize conics and make connections between graphs of conic sections and their formulas. By the end of the lesson, students will identify conic sections in multiple representations.

Focus on Disciplinary Literacy

INM

<u>Opportunities to CFU</u>
✓ What is the difference between a conic and a degenerate conic?
✓ How can you determine the orientation of a conic?

<u>Other Notes to Inform Your Planning</u>
- Students may not recall what a cross section is. The back of the student pages has images of examples of cross section for reference.
- Be sure to run the GeoGebra and Desmos activities as a student to familiarize yourself when launching this to your class.
- Students can launch Desmos on their laptops or you can project it on the board. Teacher discretion is advised.
- Student Practice is from the Desmos activity. However, if no technology is available, students have a paper copy to match conics with their formula.

Lesson Look Fors

<u>Look for teachers to...</u>
❑ Access prior knowledge of a circle and its equation in standard form. (Do Now)
❑ Uses GeoGebra to model the different types of conics.
❑ Uses Desmos to engage students in the INM and Student Practice.

<u>Look for students to...</u>
❑ Identify and explain with rationale how conics are formed.

Student Know/Do Chart

Know The coefficients of a general form equation for a conic can help determine the type and orientation of the conic.

Know The angle or direction you cut through a cone determines they type of conic you will get.

Do Identify the conic generated by the intersection of a double-napped cone and a plane (not through the vertex).

Do Identify the conic when represented in a general form equation.

Date: _____
Lesson 2: Geometry of a Parabola

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **(3.G)** Make connections between the locus definition of conic sections and their equations in rectangular coordinates.	**Necessary Materials and Pre-Lesson Prep** ▪ Graphing calculators　　▪ Document camera ▪ Desmos Activity **Lesson Structure:** 🟥 Do Now (5 min) 🟦 INM (30 min) 🟩 Student Practice (14 min) 🟪 Debrief (2 min) 🟪 Exit Ticket (4 min) **Mathematical Goal of this Lesson** This lesson begins to explore parabolas. Parabolas is not a new topics for students. In the past, students have explored characteristics of parabolas and their transformations. In this lesson, students will parabolas generated by conic sections and represent them in the standard form of equations for parabolas.	**Look for teachers to…** ❑ Use the Desmos activity to walkthrough the geometry of a parabola. **Look for students to…** ❑ Make the connection between the directrix and focus of a parabola. ❑ Defines the parabola using the directrix and focus.
Important Vocabulary ▪ **Parabola** ▪ **Definition of a Parabola** ▪ **Directrix** ▪ **Focus** ▪ Axis of Asymmetry ▪ Vertex	**Opportunities to CFU** ✓ How are the focus and directrix related?　✓ How can the focus and directrix tell you the orientation of a parabola? **Other Notes to Inform Your Planning** • Students may not recall what a cross section is. The back of the student pages has images of examples of cross section for reference. • Be sure to run the GeoGebra and Desmos activities as a student to familiarize yourself when launching this to your class. • Students can launch Desmos on their laptops or you can project it on the board. Teacher discretion is advised.	**Student Know/Do Chart** Know A parabola is the set of all points in a plane equidistant from the directrix, and the focus in the plane. Know The position of the focus and directrix helps us understand the orientation of the parabola. Do Define a parabola. Do Write equations of parabolas in standard form given a set of parameters.

Date: _____

Lesson 3: Writing and Graphing Equations of a Parabola

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **(3.G)** Make connections between the locus definition of conic sections and their equations in rectangular coordinates.	**Necessary Materials and Pre-Lesson Prep** ▪ Graphing calculators ▪ Document camera ▪ Desmos Activity Lesson Structure: 🟥 Do Now (7 min) 🟦 INM (33 min) 🟩 Student Practice (10 min) 🟪 Debrief (2 min) 🟪 Exit Ticket (3 min) **Mathematical Goal of this Lesson** This lesson continues the study of parabolas and their standard form equation. The lesson begins by recalling characteristics of parabolas learned in the previous lesson, then emphasizing how to write equations of parabolas in standard and general form.	Look for teachers to… ☐ Use the Desmos activity to walkthrough the mechanics of creating the equation for a parabola. Look for students to… ☐ Explore the relationships between the positions of the focus and the vertex as well as the directrix.
Important Vocabulary ▪ Parabola ▪ Definition of a Parabola ▪ Directrix ▪ Focus ▪ Axis of Asymmetry ▪ Vertex	**Opportunities to CFU** ✓ How are the focus and directrix related? ✓ What is the difference between standard and general form? ✓ How can the focus and directrix tell you the orientation of a parabola? **Other Notes to Inform Your Planning** • Students may not recall what a cross section is. The back of the student pages has images of examples of cross section for reference. • Be sure to run the GeoGebra and Desmos activities as a student to familiarize yourself when launching this to your class. • Students can launch Desmos on their laptops or you can project it on the board. Teacher discretion is advised.	**Student Know/Do Chart** Know — Identifying the key features of a parabola and their orientation aids in graphing and writing an equation for the parabola. Know — The standard form of an equation of a parabola gives important information about its graph and attributes. Do — Write equations of parabolas in standard form given a set of parameters. Do — Convert between standard and general forms.

Date: _____
Lesson 4: Reflective Property of Parabolas & Other Applications

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **(3.G)** Make connections between the locus definition of conic sections and their equations in rectangular coordinates.	**Necessary Materials and Pre-Lesson Prep** ▪ Graphing calculators ▪ Document camera ▪ Desmos Activity **Lesson Structure:** 🟥 Do Now (7 min) 🟦 INM (20 min) 🟩 Student Practice (20 min) 🟪 Debrief (2 min) 🟪 Exit Ticket (6 min) **Mathematical Goal of this Lesson** In this lesson, students will have opportunities to use properties of parabolas to solve real-world applications.	Look for teachers to… ❑ Use the Desmos activity to walkthrough applications of parabolas. Look for students to… ❑ Define the reflective property of a parabola. ❑ Connect real-world scenarios with a parabola as the model.
Important Vocabulary ▪ **Reflective Property of a Parabola** ▪ **Paraboloid of Revolution**	**Opportunities to CFU** ✓ How are the focus and directrix related? ✓ What is the difference between standard and general form? ✓ How can the focus and directrix tell you the orientation of a parabola? **Other Notes to Inform Your Planning** • Be sure to run the GeoGebra and Desmos activities as a student to familiarize yourself when launching this to your class. • Students can launch Desmos on their laptops or you can project it on the board. Teacher discretion is advised.	**Student Know/Do Chart** *Know* Determining characteristics of a parabola that models a real-world application allows us to solve problems. *Do* Solve real-world problems using properties of parabolas.
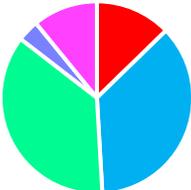
Date: _____
Lesson 5: Geometry of an Ellipse

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **(3.H)** Use the characteristics of an ellipse to write and apply the equation of an ellipse with center (h, k).	**Necessary Materials and Pre-Lesson Prep** ▪ Graphing calculators ▪ Document camera ▪ Desmos Activity Lesson Structure: ■ Do Now (5 min) ■ INM (25 min) ■ Student Practice (15 min) ■ Debrief (5 min) ■ Exit Ticket (5 min) **Focus on Disciplinary Literacy** — INM **Mathematical Goal of this Lesson** In this lesson, we will explore the key characteristics of an ellipse by constructing it using technology. The Desmos activity will walkthrough the geometry of an ellipse and use it to formally define an ellipse.	Look for teachers to… ❑ Use the Desmos activity to walkthrough the geometry of an ellipse. Look for students to… ❑ Explore the relationships between the positions of the foci and locus of points that generate an ellipse.
Important Vocabulary ▪ Ellipse ▪ Standard Forms of the Equation of an Ellipse ▪ Foci ▪ Major Axis ▪ Minor Axis ▪ Center ▪ Vertices ▪ Co-Vertices	**Opportunities to CFU** ✓ What is the relationship between the points on the ellipse and the foci? ✓ How can you tell if an ellipse is horizontally or vertically oriented? **Other Notes to Inform Your Planning** • Be sure to run the GeoGebra and Desmos activities as a student to familiarize yourself when launching this to your class. • Students can launch Desmos on their laptops or you can project it on the board. Teacher discretion is advised. • In the Desmos activity, r_x and r_y imply the "radii" along the horizontal and vertical axes of the ellipse, respectfully.	**Student Know/Do Chart** Know — Identifying the key features of an ellipse and their orientation aids in graphing and writing an equation for the ellipse. Know — The standard form of an equation of an ellipse gives important information about its graph and attributes. Do — Define an ellipse. Do — Write equations of ellipses in standard form given a set of parameters.

Date: _____
Lesson 6: Writing and Graphing Equations of an Ellipse

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **(3.H)** Use the characteristics of an ellipse to write and apply the equation of an ellipse with center (h, k).

Important Vocabulary

- Ellipse
- Standard Forms of the Equation of an Ellipse
- Foci
- Major Axis
- Center
- Minor Axes
- Vertices
- Co-Vertices
- **Sum of Focal Radii**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep
- Graphing calculators
- Desmos Activity
- Document camera

Lesson Structure:
- 🟥 Do Now (10 min)
- 🟦 INM (20 min)
- 🟩 Student Practice (15 min)
- 🟪 Debrief (5 min)
- 🟧 Exit Ticket (5 min)

Mathematical Goal of this Lesson
In the previous lesson, students learned how the standard form of an equation for an ellipse determines its characteristics. In this lesson, students will convert general for equations to standard form equations, determine equations for ellipses given defining characteristics, and graph ellipses.

Opportunities to CFU
✓ Given an equation in standard form, how can you tell it's an ellipse?

✓ Given an equation in general form, how can you tell it's an ellipse?

Other Notes to Inform Your Planning
- Be sure to run the GeoGebra and Desmos activities as a student to familiarize yourself when launching this to your class.
- Students can launch Desmos on their laptops or you can project it on the board. Teacher discretion is advised.
- Consider reviewing completing the square technique before the Do Now or replacing parts of the Do Now.

Lesson Look Fors

Look for teachers to...
❑ Use the Desmos activity to walkthrough the geometry of an ellipse.

Look for students to...
❑ Leverage Desmos to model ellipses in vertical and horizontal orientations.
❑ Identifying characteristics of an ellipse given some parameters.

Student Know/Do Chart

Know — Identifying the key features of an ellipse and their orientation aids in graphing and writing an equation for the ellipse.

Know — The standard form of an equation of an ellipse gives important information about its graph and attributes.

Do — Write equations of ellipses in standard form given a set of parameters.

Do — Convert between standard and general forms.

Date: _____
Lesson 7: Reflective Property of Ellipses & Other Applications

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **(3.H)** Use the characteristics of an ellipse to write and apply the equation of an ellipse with center (h, k).	**Necessary Materials and Pre-Lesson Prep** ▪ Graphing calculators ▪ Document camera ▪ Desmos Activity **Lesson Structure:** 🟥 Do Now (7 min) 🟦 INM (20 min) 🟩 Student Practice (20 min) 🟪 Debrief (3 min) 🟪 Exit Ticket (5 min) **Mathematical Goal of this Lesson** This lesson applies the skills learned in the last two lessons regarding ellipses to contextual situations. The Do-Now asks students to determine several characteristics of an ellipse given its equation. The INM uses a Desmos activity to visualize some applications of ellipses. The ellipsoid of revolution and its reflective properties are introduced and used in context. Students are expected to find characteristics of ellipses, find and apply the equations of ellipses that model the situation. **Opportunities to CFU** ✓ How do you find the distance between the foci? ✓ How do you find a and b given the major and minor axes? **Other Notes to Inform Your Planning** • Be sure to run the GeoGebra and Desmos activities as a student to familiarize yourself when launching this to your class. • Students can launch Desmos on their laptops or you can project it on the board. Teacher discretion is advised. • Consider reviewing completing the square technique before the Do Now or replacing parts of the Do Now.	Look for teachers to... ❑ Use the Desmos activity to walkthrough the applications of an ellipse. Look for students to... ❑ Identify characteristics of an ellipse given certain attributes. ❑ Explain how some phenomena can be modeled by an elliptical curve. **Student Know/Do Chart** Know Writing and applying equations of ellipses can help solve real world problems. Know Characteristics of an ellipse can be used to determine a modeling equation. Do Solve real-world problems using equations of ellipses.
Important Vocabulary ▪ Ellipse ▪ Standard Forms of the Equation of an Ellipse ▪ Foci ▪ Major Axis ▪ Center ▪ Minor Axes ▪ Vertices ▪ Co-Vertices ▪ Sum of Focal Radii		

Date: _____
Lesson 8: Geometry of a Hyperbola

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **(3.I)** Use the characteristics of a hyperbola to write and apply the equation of a hyperbola with center (h, k).

Important Vocabulary

- **Definition of a Hyperbola**
- **Standard Forms of the Equation of a Hyperbola**
- Focal Axis
- Center
- Foci
- Vertices
- **Transverse Axis**
- **Conjugate Axis**
- Asymptotes

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep
- Graphing calculators
- Desmos Activity
- Document camera

Lesson Structure:
- 🟥 Do Now (5 min)
- 🟦 INM (30 min)
- 🟩 Student Practice (12 min)
- 🟪 Debrief (3 min)
- 🟪 Exit Ticket (5 min)

Focus on Disciplinary Literacy

INM

Mathematical Goal of this Lesson
In this lesson, students will explore the relationships between the distances on a hyperbola to the foci that generates the curve. Desmos is used in the INM to walkthrough key characteristics of a hyperbola, scale, location, and orientation. Students will be able to define the hyperbola as the locus of points on a plane whose distance from two fixed points have a constant difference.

Opportunities to CFU
✓ What should the difference in distances between the foci and a point on the curve be?
✓ How can you determine the orientation of the hyperbola given in standard form?

Other Notes to Inform Your Planning
- Be sure to run the GeoGebra and Desmos activities as a student to familiarize yourself when launching this to your class.
- Students can launch Desmos on their laptops or you can project it on the board. Teacher discretion is advised.
- Card sort activity can be done without tech, if you choose to not have students use Desmos individually.

Lesson Look Fors

Look for teachers to...
☐ Use the Desmos activity to walkthrough the geometry of a hyperbola.

Look for students to...
☐ Make conjectures about the distances between the foci and a point on the curve.
☐ Discuss the definition of a hyperbola.
☐ Explain how the orientation of the hyperbola can be defined.

Student Know/Do Chart

Know 🎓 Identifying the key features of a hyperbola and their orientation aids in graphing and writing an equation for the hyperbola.

Know 🎓 The standard form of an equation of a hyperbola gives important information about its graph and attributes.

Do 🎯 Define a hyperbola.

Do 🎯 Write equations of hyperbolas in standard form given a set of parameters.

Date: _____
Lesson 9: Writing and Graphing Equations of a Hyperbola

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **(3.I)** Use the characteristics of a hyperbola to write and apply the equation of a hyperbola with center (h, k).	**Necessary Materials and Pre-Lesson Prep** ▪ Graphing calculators ▪ Document camera ▪ Desmos Activity **Lesson Structure:** 🟥 Do Now (7 min) 🟦 INM (25 min) 🟩 Student Practice (15 min) 🟪 Debrief (3 min) 🟪 Exit Ticket (5 min) **Mathematical Goal of this Lesson** In the previous lesson, students learned how the standard form of an equation for a hyperbola determines its characteristics. In this lesson, students will convert general form equations to standard form equations, determine equations for hyperbolas given defining characteristics, and graph hyperbolas.	**Look for teachers to...** ❑ Use the Desmos activity to walkthrough the graphs and characteristics of hyperbolas. **Look for students to...** ❑ Leverage Desmos to model hyperbolas in vertical and horizontal orientations. ❑ Identifying characteristics of a hyperbola given some parameters.

Important Vocabulary
▪ Definition of a Hyperbola
▪ Standard Forms of the Equation of a Hyperbola
▪ Focal Axis
▪ Center
▪ Foci
▪ Vertices
▪ Transverse Axis
▪ Conjugate Axis
▪ Asymptotes
	Opportunities to CFU
	✓ How can the characteristics of a hyperbola be identified from an equation in standard form?
	✓ How can the general form equation for a hyperbola be transformed into the standard form?
	✓ How can the equation for a hyperbola be determined from its characteristics?
	
	Other Notes to Inform Your Planning
	• A Desmos Activity is used throughout the Do-Now and the INM to help students visualize the concepts as well as to foster more independent work by providing students with self-checking slides.
	• The Do-Now practices previously learned concepts:
	    ○ completing the square for ellipse equations
	    ○ identifying the type of conic section represented by a general form equation
	• The INM first covers the process of completing the square for hyperbola equations, then the process of determining the standard form equation of a hyperbola when given defining characteristics.
	• The Student Practice and Extra Practice problems give students more opportunity to master the objective of the lesson.
		Student Know/Do Chart
		Know Identifying the key features of a hyperbola and their orientation aids in graphing and writing an equation for the hyperbola.
		
		Know The standard form of an equation of a hyperbola gives important information about its graph and attributes.
		
		Do Write equations of hyperbolas in standard form given a set of parameters.
		
		Do Convert between standard and general forms.

Date: _____
Lesson 10: Reflective Property of Hyperbolas & Other Applications

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆(3.I) Use the characteristics of a hyperbola to write and apply the equation of a hyperbola with center (h, k).	**Necessary Materials and Pre-Lesson Prep** ▪ Graphing calculators ▪ Document camera ▪ Desmos Activity Lesson Structure: 🟥 Do Now (7 min) 🟦 INM (25 min) 🟩 Student Practice (15 min) 🟪 Debrief (3 min) 🟪 Exit Ticket (5 min) **Mathematical Goal of this Lesson** This lesson wraps up the concepts and applications of the hyperbola. Students will explore how certain phenomena can be modeled by a hyperbolic curve and apply the reflective property in contextual settings. **Opportunities to CFU** ✓ What is significant about the foci of a hyperbola? **Focus on Disciplinary Literacy** — INM **Other Notes to Inform Your Planning** • A Desmos activity accompanies all problems in the Do Now and the INM to encourage independent work and to help with visualization of applications. • The Do Now reviews writing the equation for a hyperbola, solving for the value of one variable in a hyperbola when given the value of the other variable, the definition of the hyperbola and the relationship between equation parameters and that definition. • The INM introduces and applies the reflective property of hyperbolas as well as other applications of hyperbolas and hyperboloids. • Student Practice and Extra Practice problems give more opportunity to apply properties of hyperbolas in context.	**Look for teachers to…** ❑ Use the Desmos activity to walkthrough the applications of a hyperbola. **Look for students to…** ❑ Identify characteristics of a hyperbola given certain attributes. ❑ Explain how some phenomena can be modeled by a hyperbolic curve. **Student Know/Do Chart** **Know** Writing and applying equations of hyperbolas can help solve real world problems. **Know** Characteristics of a hyperbola can be used to determine a modeling equation. **Do** Solve real-world problems using equations of hyperbolas.
Important Vocabulary ▪ Definition of a Hyperbola ▪ Standard Forms of the Equation of a Hyperbola ▪ Focal Axis ▪ Center ▪ Foci ▪ Vertices ▪ Transverse Axis ▪ Conjugate Axis ▪ Asymptotes ▪ **Reflective Property of a Hyperbola** ▪ **Hyperboloid of Revolution**		

Date: _____
Lesson 11: Eccentricity and Review

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **(3.G)** Make connections between the locus definition of conic sections and their equations in rectangular coordinates.	**Necessary Materials and Pre-Lesson Prep** ▪ Graphing calculators ▪ Document camera ▪ Desmos Activity Lesson Structure: 🟥 Do Now (10 min) 🟦 INM (20 min) 🟩 Student Practice (15 min) 🟪 Debrief (2 min) 🟪 Exit Ticket (5 min) **Mathematical Goal of this Lesson** This lesson reviews all conic forms and introduces eccentricity. In the INM, students will be able to connect eccentricity with the associated conic section. **Opportunities to CFU** ✓ How can you tell the conic form given eccentricity? ✓ How do you calculate eccentricity? **Other Notes to Inform Your Planning** • The Do-Now is a Desmos card sort that reviews definitions of the conic sections and their reflective properties as well as the general form and standard form of each. • The INM uses a Desmos slide to explore the connection between eccentricity and the type and shape of a conic section. Then problems are done to use eccentricity while reviewing the skills learned in the unit regarding writing and graphing equations for conic sections. Desmos slides are uses so that students can self-check their work. • The Student Practice and Extra Practice sections give students more practice and review.	**Look for teachers to…** ❑ Use the Desmos activity to show the relationship between conic section and eccentricity. **Look for students to…** ❑ Examine and discuss how eccentricity generates conic sections. ❑ Identifying the type of conics section given eccentricity. **Student Know/Do Chart** *Know* Identifying the key features of a conic section aids in graphing and writing an equation for the conic section. *Know* The standard form of an equation of a conic section gives important information about its graph and attributes. *Know* The eccentricity of a conic section determines its type. *Do* Compute and interpret eccentricity.
Important Vocabulary ▪ Eccentricity		

To review <mark>topics based on your data on Success Days</mark>, a good source for additional resources is the Khan Academy Conic Sections Unit. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
(3.H) Use the characteristics of an ellipse to write and apply the equation of an ellipse with center (h, k).	- Ellipse - Standard forms of the equation of an ellipse - Foci - Major axis - Center - Minor axes - Vertices - Co-vertices - Sum of focal radii	<u>Precalculus TEKS</u> **(3.H)** use the characteristics of an ellipse to write and apply the equation of an ellipse with center (h, k) <u>Algebra 2 TEKS (not ellipse, but involves a conic)</u> **(4.B)** write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening **AP Calculus AB 2015 Released FRQ #6** *This free-response problem does not contain conics. However, students work with implicitly defined functions in precalculus.* **AP® CALCULUS AB** **2015 SCORING GUIDELINES** **Question 6** Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\dfrac{dy}{dx} = \dfrac{y}{3y^2 - x}$. (a) Write an equation for the line tangent to the curve at the point $(-1, 1)$. (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical. (c) Evaluate $\dfrac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$. (a) $\left.\dfrac{dy}{dx}\right\|_{(x,y)=(-1,1)} = \dfrac{1}{3(1)^2 - (-1)} = \dfrac{1}{4}$ $2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$ An equation for the tangent line is $y = \dfrac{1}{4}(x+1)+1$. (b) $3y^2 - x = 0 \Rightarrow x = 3y^2$ $3 : \begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$ So, $y^3 - xy = 2 \Rightarrow y^3 - (3y^2)(y) = 2 \Rightarrow y = -1$ $(-1)^3 - x(-1) = 2 \Rightarrow x = 3$ The tangent line to the curve is vertical at the point $(3, -1)$.

VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Geometry	Pre-Calculus	AP Calculus AB/BC
• **G.12E** Show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center $(h,\ k), (x-h)^2 + (y-k)^2 = r^2$.	• **(3.F)** Determine the conic section formed when a plane intersects a double-napped cone. • **(3.G)** Make connections between the locus definition of conic sections and their equations in rectangular coordinates. • **(3.H)** Use the characteristics of an ellipse to write the equation of an ellipse with center (h, k). • **(3.I)** Use the characteristics of a hyperbola to write the equation of a hyperbola with center (h, k).	• **CHA-5.B.1** Volumes of solids with square and rectangular cross sections can be found using definite integrals and the area formulas for these shapes. • **CHA-5.B.2** "… triangular cross sections …" • **CHA-5.B.3** "… semicircular and other geometrically defined cross sections …" • **CHA-5.C.1** Volumes of solids of revolution around the x- or y-axis may be found by using definite integrals with the disc method. • **CHA-5.C.2** Volumes of solids of revolution around any horizontal or vertical line in the plane may be found by using definite integrals with the disc method. • **CHA-5.C.3** Volumes of solids of revolution around the x- or y-axis whose cross sections are ring shaped may be found using definite integrals with the washer method. NOTE: These "essential knowledge" (EK) standards are from The College Board Concept Outline for AP Calculus AB/BC, not the TEKS.
Algebra 2		
• **2A.4B** Write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening.		